Statistical Performance Sensitivity - a Valuable Measure for Manufacturing Oriented CAD

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ABSTRACT

This paper delineates the random and deterministic components of the manufacturing and design process. Design for manufacture includes determining the deterministic parameter vector, P_o, such that some manufactured unit performance statistic, like $E\{G(P_o+\Delta\mathcal{P})\}$, or Yield($P_o$), is controlled or optimized, where $G(\cdot)$ is a nonlinear function describing performance. The statistical sensitivities are an extension of classic sensitivities, applied to the general statistical outcome of the manufacturing process, and we develop an efficient way to calculate them, using Monte Carlo estimation.

The original contributions of this paper are:

1) The careful definition of variables, both deterministic and random, and their representation in manufacturing.
2) The general definition of statistical sensitivity to include performance statistics and yield.
3) The efficient estimation of all the statistical sensitivities by using one Monte Carlo simulation.
4) The application of these sensitivities and their calculation to a Salin and Key active filter.

INTRODUCTION

This paper address one problem of manufacturing oriented computer aided design, meaningful sensitivity measures. Our viewpoint is that a manufacturing oriented design aims at the collection (ensemble) of 1000's of manufactured, assembled, and tested units. The ultimate result of manufacturing is not any single unit or any single unit performance, but an ensemble of manufactured units and unit performances. The goal of manufacturing oriented design is to control and optimize some property of the ensemble of manufactured units; ensemble properties include average performance, performance variance, and parametric yield, which is the percentage of units passing performance test specifications. This paper is most similar to [1,2], except here the definitions are more general, and the calculation involves Monte Carlo estimation, rather than analysis using truncated series. We used this calculation technique in [3], but only with respect to yield sensitivity.

THE DESIGN AND MANUFACTURING ENVIRONMENTS

It is important to understand the random and deterministic parts of the design and manufacturing processes. Therefore we define:

Unit, the intended element to be manufactured; like a microwave amplifier, MMIC low noise mixer, or an entire communications system.

Parameter, $P = (p_1, p_2, \ldots, p_n)$, the design variables which describe the unit, for instance circuit parameters would be the values of the resistors, inductors and capacitors, and system parameters would be the block gains, bandwidths and noise figures. This assumes the unit structure and manufacturing technology have been chosen.

Performance, $G(P) = (g_1, g_2, \ldots, g_k)$, the unit performance, which can be gain and noise figure for an amplifier, or signal to noise and input sensitivity for a communications system. CAD based simulation software generally calculates $G(P)$ for a given value of P.

Design Parameter, $P_o = (p_{1o}, p_{2o}, \ldots, p_{no})$, the parameters that are specified by the design engineer and are the main result of the CAD oriented part of the design process.

Nominal Design Performance, $G_o = G(P_o)$, the unit performance at the design parameter values.

Note: script variables denote random variables

Unit Parameter, $\mathcal{P} = (\rho_1 \rho_2 \ldots \rho_n)$, the parameter vector that an individual unit encounters during manufacturing.

Parameter Density, $f_{\mathcal{P}}(P)$, the joint probability density function that describes the parameter values, $\mathcal{P}$, used during manufacturing. We assume in this work that $f_{\mathcal{P}}(P)$ can be mathematically approximated and that

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it is time invariant.

Unit Performance, $\mathfrak{b} = G(\mathcal{P})$, the performance of a manufactured unit. This is a random variable because the parameter vector is random during manufacturing. Note: nonlinear transformations of random vectors are difficult to statistically quantify.

Performance Statistic, a characteristic of the ensemble performance encountered during manufacturing; like mean, $E(\mathfrak{b})$; variance, $E(\mathfrak{b} - E(\mathfrak{b}))^2$; or parametric yield, Y, which is the fraction of units which meets performance specifications during manufacturing (as the number of units manufactured gets very large). "E" denotes expected value.

DESIGN AND MANUFACTURING VARIABLES

The unit parameter vector is modeled as $\mathcal{P} = P_o + \Delta\mathcal{P}$, where P_o is the design parameter, and $\Delta\mathcal{P}$ is a random variable describing the manufacturing environment. $f_{\mathcal{P}}(P)$ is the unit parameter density; $f_{\mathcal{P}}(P) = f_{\Delta\mathcal{P}}(P) * \delta(P - P_o)$, $\delta(P - P_o)$ is the n-dimension dirac delta function, * is n-dimension convolution and $f_{\Delta\mathcal{P}}(\Delta P)$ is the joint parameter density of the manufacturing variation $\Delta\mathcal{P}$. A presentation of many of these relations is given in Figure 1, where our manufacturing model is described somewhat graphically.

THREE SENSITIVITY DEFINITIONS

There are three sensitivity definitions of interest in this paper:

Manufacturing Sensitivity - A unit's actual performance statistic change during manufacturing as a function of the change in a design parameter when the unit is actually manufactured. This serves as the conceptual goal for our manufacturing oriented sensitivity analysis

Classic Sensitivity - A unit's calculated change in performance, G(P), as a function of the change in a parameter, p_i.

Statistical Sensitivity - A Unit's calculated change in performance statistic, like $E(\mathfrak{b})$, as a function of the change in a design parameter, p_{io}.

Statistical Sensitivity comes closest to describing the idea of Manufacturing Sensitivity. In fact using Classic Sensitivity to infer the properties of Manufacturing Sensitivity can lead to poor results as indicated by the brinksmanship behavior [7,9].

We note that these definitions are given in the form of single point sensitivities, but each can be modified to include the concepts of Large Change, Multiparameter and Large Change Multiparameter Sensitivities. [4,5].

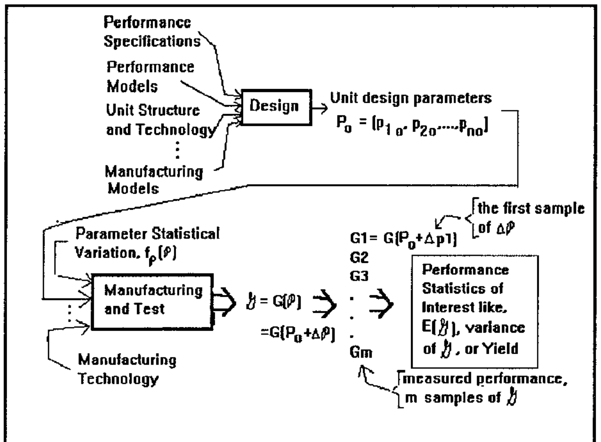

Figure 1 - Graphical description of the design and manufacturing process.

STATISTICAL SENSITIVITY

Two forms of statistical sensitivity that we investigate in this paper are:

Statistical Performance Sensitivity:

(note: With no loss of generality, we'll let statistical performance be $E(\mathfrak{b})$).

$$S_{p_{io}}^{E(\mathfrak{b})} = \frac{d}{dp_{io}} \int_{-\infty}^{\infty} G(P) f_{\mathcal{P}}(P)\, dP$$

and

Yield Sensitivity:

$$S_{p_{io}}^{Y(P_o)} = \frac{d}{dp_{io}} \int_{-\infty}^{\infty} accept(P) f_{\mathcal{P}}(P)\, dP$$

recalling that $\mathcal{P} = P_o + \Delta\mathcal{P}$.

The accept function, accept(P), is a binary function which equals one if G(P) meets all performance test specifications, otherwise it is zero.

CALCULATING STATISTICAL SENSITIVITY

The key to efficiently estimating the statistical sensitivities is the performance and the yield factors. They are given as

$$\bar{\mathfrak{b}}(p_{io}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} G(P) f_{\mathcal{P}}(p_1, \ldots, p_{i-1}, p_{io}, p_{i+1}, \ldots, p_n)\, dp_1 \ldots dp_{i-1} dp_{i+1} \ldots dp_n.$$

$$Y(p_{io}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} accept(P) f_{\mathcal{P}}(p_1, \ldots, p_{i-1}, p_{io}, p_{i+1}, \ldots, p_n)\, dp_1 \ldots dp_{i-1} dp_{i+1} \ldots dp_n.$$

These factors are essentially the statistical averages with all the parameter values changing according to their statistical descriptions, except the ith parameter, which is held constant.

To better calculate the performance and yield factors, we will define unbiased estimators $\hat{\mathcal{D}}(p_{io})$ and $\hat{Y}(p_{io})$,

$$\hat{\mathcal{D}}(p_{io}) = \frac{1}{M}\sum_{i=1}^{M} G(P_{io})$$

$$\hat{Y}(p_{io}) = \frac{1}{M}\sum_{i=1}^{M} accept(P_{io})$$

Where Pio is a parameter vector chosen according to $f_{\rho}(P)\delta(p_i - p_{io})$, properly normalized.

We can develop an approximation to the statistical factors by performing a Monte Carlo analysis in which all of the parameters are allowed to vary according to $f_{\rho}(P)$. Each parameter range is divided into equal intervals called "bins", and the "binned" statistical factors for each parameter are calculated separately for each bin. A plot of the binned approximation of the statistical factors versus the p_{io} bin center values is called a statistical factor histogram [6,7]. This technique estimates all statistical factors with one M-point Monte Carlo simulation, independent of the number of design parameters. Essentially the statistical sensitivity is estimated by measuring the slope of the Factor Histogram, as illustrated in Figure 2.

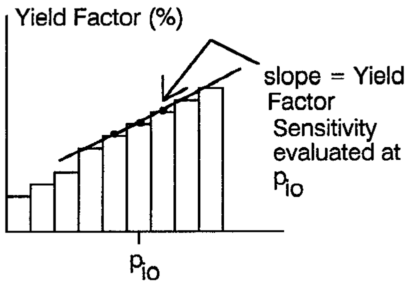

Figure 2. Yield Factor Sensitivity as the slope of the yield factor histogram.

Since these sensitivities are written in terms of the difference of two estimated values, the errors in this calculation can be large. A careful error analysis is recommended. Based on a 1000 point Monte Carlo simulation, we have fit lines through all the bins, using a least squares fit, and then assigned the line slope as the parameter sensitivity [3]. This has shown good results, although a complete statistical error analysis is not presently available.

EXAMPLE

Salen and Key Filter

The example is a Salen and Key band pass filter which is used often in the statistical design literature [8]. The circuit configuration is shown in Figure 3, and the nominal parameter values and their tolerances are given in Table 1.

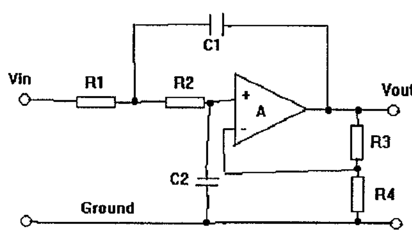

Figure 3- Salen and Key filter configuration.

parameter	p_o	tolerance
R1	1K Ohm	1%
R2	2.5 K Ohms	1%
R3	4 K Ohms	1%
R4	12K Ohm	1%
C1	1uF	.02%
C2	.1uF	.02%
A	3000	none

Table 1 - Nominal values and tolerances for the Salen and Key filter

The Filter Q is used as the performance, and it is mathematically described as

$$Q = \frac{(R_1R_2C_1C_2)^{-\frac{1}{2}}}{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} - \frac{R_3}{R_2R_4C_2}\right)}$$

The Q at the nominal parameter values is 19.0. The statistical model uses uniform independent parameters with the extent of the parameter variation being twice (i.e. $\pm$) the stated tolerance. The specification for the circuit is

10 <= Q <= 40.

The estimated yield is .887 with a 95% confidence of less than $\pm$1% with a 10,000 point Monte Carlo simulation.

At the nominal value given in Table 1, we calculated the yield factor, the average performance factor, and the performance standard deviation factor. Each of these is given in Figures 4, 5, and 6 respectively.

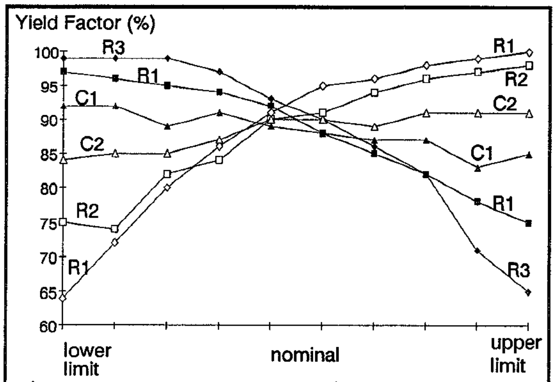

Figure 4 - Plot of estimated Yield Factor versus parameter value for the Salen and Key filter, evaluated at Po given in Table 1.

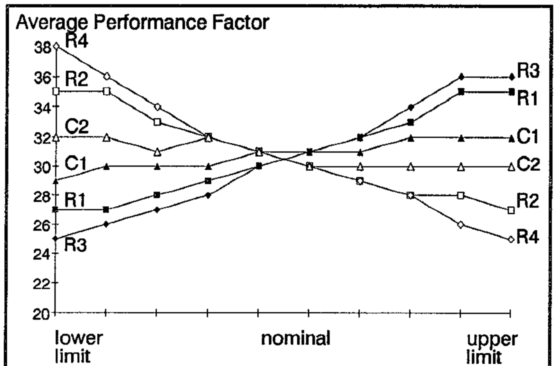

Figure 5 - Plot of estimated Average Performance Factor versus parameter value, evaluated at Po given in Table 1.

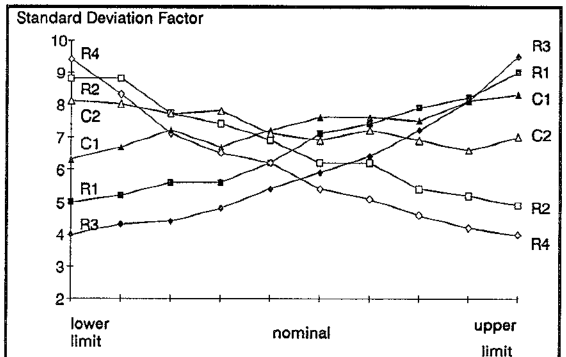

Figure 6 - Plot of the estimated Performance Standard Deviation Factor versus parameter value, evaluated at Po given in Table 1.

The slopes of these plots, evaluated at the appropriate parameter value, are the statistical sensitivities. For example, for R1 the Yield Sensitivity is -20%(yield) per 1%(R), the Average Performance Sensitivity is +5(Q) per 1%(R) and the Standard Deviation (S.D.) Sensitivity is +2(S.D.) per 1%(R). All the statistical data displayed in these figures was made from a single 10,000 point Monte Carlo analysis of the circuit. The ratio of the Average Performance Factor and the Standard Deviation Factor can create a type of "Signal to Noise" factor that is presently popular in the "Quality" literature.

CONCLUSIONS

The model introduced in this paper shows, from our viewpoint, the differences between statistical sensitivity and classic sensitivity. Our statistical sensitivities are an extension of the classic sensitivities, applied to the general statistical outcome of the manufacturing process. The calculation procedure given for these sensitivities is general, and many different statistical sensitivities, for all parameters in the design, can be estimated from one Monte Carlo simulation of the unit.

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